Summer Packet

AP Calculus BC

Summer 2022

Enclosed is a packet that needs to be completed this summer. You may use any resources that are available to you. Some questions may require the use of a calculator. I will collect it the first day of class in September, and I will grade it. This will be your first Calculus grade.

Enjoy your summer. See you in the Fall.

Mr. A

Sequences

Example #1:
List the 1st five terms of the following sequences:

a) $\{a_n\} = \{3 + (-1)^n\}$

b) $\{a_n\} = \left\{\dfrac{2n}{1+n}\right\}$

c) $\{a_n\} = \left\{\dfrac{n^2}{2^n - 1}\right\}$

Example #2

Find the limit of the sequence whose *n*th term is: $a_n = \left(1 + \dfrac{1}{n}\right)^n$

Example #3
Determine whether the following sequences converge or diverge:

a) $\{a_n\} = \{3 + (-1)^n\}$

b) $\{a_n\} = \left\{\frac{n}{1-2n}\right\}$

Example #4
Show that the sequence whose *n*th term is $\{a_n\} = \left\{\frac{n^2}{2^n - 1}\right\}$ converges.

Example #5
Recall:

Does the sequence $\{a_n\} = \left\{\frac{(-1)^n}{n!}\right\}$ converge?

Example #6

Find a sequence whose first 5 terms are $\frac{2}{1},\frac{4}{3},\frac{8}{5},\frac{16}{7},\frac{32}{9}$........ Does it converge?

Example #7

Determine the *n*th term of the sequence whose first five terms are:

$-\frac{2}{1},\frac{8}{2},-\frac{26}{6},\frac{80}{24},-\frac{242}{120},$........ Then decide whether it converges or diverges.

Monotonic Sequences and Bounded Sequences

Definition of a Monotonic Sequence:

Example #8

Determine whether each sequence with the given *n*th term is monotonic:

a) $\{a_n\} = \{3 + (-1)^n\}$ b) $\{a_n\} = \left\{\dfrac{2n}{1+n}\right\}$ c) $\{a_n\} = \left\{\dfrac{n^2}{2^n - 1}\right\}$

Definition of a Bounded Sequence:

THEOREM:

Example #9

Discuss whether the following sequences are bounded and/or monotonic.

a) $\{a_n\} = \left\{\dfrac{1}{n}\right\}$ b) $\{a_n\} = \left\{\dfrac{n^2}{(n+1)}\right\}$ c) $\{a_n\} = \{(-1)^n\}$

H.W. 4.1 – p.555-556 #13-16,33,34,37, 38,42,51-55,57,60,61,71,73

Series and Convergence

Infinite Series:

Definition of Convergent or Divergent Series

Example #1

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

Example #2

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}-\frac{1}{n+1}\right)$$

Example #3

$$\sum_{n=1}^{\infty}1$$

Example #4

Find the sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{2}{4n^2-1}$.

Geometric Series:

Convergence of a Geometric Series

Example #5

Test if the following geometric sequence converges $\sum_{n=0}^{\infty} \frac{3}{2^n}$. If it does, find its sum.

Example #6

Test if the following geometric sequence converges $\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$. If it does, find its sum.

Example #7

Use a Geometric Series to write $.0808\overline{08}$ as a fraction.

_n_th-Term Test for Divergence

Limit of nth Term of a Convergent Sequence

nth-Term Test for Divergence

Example #8
Examine the following series:

a) $\sum_{n=0}^{\infty} 2^n$

b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$

c) $\sum_{n=1}^{\infty} \frac{1}{n}$

Example #9
A ball is dropped from a height of 6 feet and begins bouncing. The height of each bounce is three-fourths the height of the previous bounce. Find the total vertical distance travelled by the ball.

H.W. 4.2 – p.564-565 #7-21odd, 31-41odd, 49,55,57

The Intergral Test and *p*-series

The Integral Test

Example #1

Apply the Integral Test to the series $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$.

Example #2

Apply the Integral Test to the series $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$.

p-Series and Harmonic Series

Example #3
Do the following p-series converge or diverge?

a. $\sum_{n=1}^{\infty} \frac{1}{n}$

b. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

Example #4
Does the following series converge or diverge?

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n}$$

H.W. 4.3 – p.571 #1–7odd, 15-21odd

Comparisons of Series

Direct Comparison Test

Example #1

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{1}{2+3^n}$.

Example #2

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$.

Limit Comparison Test

Example #3
Show that the following harmonic series diverges:

$$\sum_{n=1}^{\infty} \frac{1}{an+b}, \quad a>0, \quad b>0$$

RECOGNITION:

Example #4

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$.

Example #5

Determine convergence or divergence of $\sum_{n=1}^{\infty} \frac{n2^n}{4n^3+1}$.

H.W. 4.4 – p.578 #3–23odd, 29-35odd

Alternating Series

Example #1

Determine the convergence or divergence of $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$.

Example #2

Determine the convergence or divergence of $\sum_{n=1}^{\infty}\frac{n}{(-2)^{n-1}}$.

Example #3

a) Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(n+1)}{n}$.

Alternating Series Remainder

Example #4
Approximate the sum of the following series by its first six terms.

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n!}$$

Absolute and Conditional Convergence

Example #5
Determine whether each series is convergent or divergent. Classify any convergent series as absolutely or conditionally convergent.

a) $\sum_{n=1}^{\infty} \frac{(-1)^{n(n+1)/2}}{3^n}$

b) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n!}{2^n}$

d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

H.W. 4.5 – p.587 #9–25odd, #37-41odd

Ratio and Root Tests

The Ratio Test

Example #1

Determine the convergence or divergence of $\sum_{n=0}^{\infty} \frac{2^n}{n!}$

Example #2
Determine whether each of the following series converge or diverge.

A) $\sum_{n=1}^{\infty} \frac{n^2 2^{n+1}}{3^n}$

B) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

Example #3

Determine the convergence or divergence of $\sum_{n=1}^{\infty} (-1)^n \frac{\sqrt{n}}{n+1}$.

The Root Test

Example #4

Determine the convergence or divergence of $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$.

Guidelines for Testing a Series for Convergence or Divergence

1. Does the *n*th term approach 0? If not, series diverges.
2. Is the series one of the special types – geometric, *p*-series, telescoping, or alternating?
3. Can the Integral Test, the Root test or the Ratio test be applied?
4. Can the series be compared favorably to one of the special types?

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Example #5

Determine the convergence or divergence of the following series.

a. $\sum_{n=1}^{\infty} \frac{n+1}{3n+1}$ b. $\sum_{n=1}^{\infty} \left(\frac{\pi}{6}\right)^n$ c. $\sum_{n=1}^{\infty} ne^{-n^2}$ d. $\sum_{n=1}^{\infty} \frac{1}{3n+1}$

e. $\sum_{n=1}^{\infty} (-1)^n \frac{3}{4n+1}$ f. $\sum_{n=1}^{\infty} \frac{n!}{10^n}$ g. $\sum_{n=1}^{\infty} \left(\frac{n+1}{2n+1}\right)^n$

H.W. 4.6 – p.594-595
#11–23odd, #33-51odd

Taylor Polynomials and Approximations

Polynomial Approximations of Elementary Functions

Example #1
For the function $f(x) = e^x$, find a first-degree polynomial function, $P_1(x) = a_0 + a_1 x$, whose value and slope agree with the value of *f* at *x* = 0.

Example #2
Construct a table comparing the values of the polynomial
$P_3(x) = 1 + x + \frac{1}{2}x^2 + \frac{1}{3!}x^3$ with $f(x) = e^x$ for several values near 0.

Taylor and Maclaurin Polynomials

So the *n*th Maclaurin polynomial for $f(x) = e^x$ is given by:

Example #3
Find the Taylor polynomials $P_0, P_1, P_2, P_3, \text{ and } P_4$ for $f(x) = \ln x$ centered at c = 1.

Example #4

Find the Maclaurin Polynomials $P_0, P_{2,} P_{4,} and\ P_6$ for $f(x) = \cos x$. Use P_6 to approximate the value of cos(0.1).

Example #5

Find the 3rd Taylor polynomial for $f(x) = \sin x$ expanded about c = $\dfrac{\pi}{6}$.

Remainder of a Taylor Polynomial

Example #6

The third Maclaurin polynomial for sin *x* is given by $P_3(x) = x - \frac{x^3}{3!}$. Use Taylors Theorem to approximate sin(0.1) by $P_3(0.1)$ and determine the accuracy of the approximation.

Example #7

Determine the degree of the Taylor polynomial $P_n(x)$ expanded about $c = 1$ that should be used to approximate ln(1.2) so that the Lagrange Error is less than 0.001.

H.W. 4.7 – p.604-605
#9,11,15,17,19,33,34,37

Power Series

Examples of Power Series:

a.

b.

c.

Radius and Interval of Convergence

Examples:

1) Find the radius of convergence of $\sum_{n=0}^{\infty} n!x^n$.

2) Find the radius of convergence of $\sum_{n=0}^{\infty} 3(x-2)^n$.

3) Find the radius of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$.

Endpoint Convergence

Examples:

4) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n}$.

5) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{(-1)^n (x+1)^n}{2^n}$.

6) Find the interval of convergence of $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$.

H.W. 4.8 – p.613-614
#1,3,5,7,9,11,15,16,17

Intervals of Convergence for $f(x)$, $f'(x)$ and $\int f(x)$.

Example

Given $f(x)=\sum_{n=1}^{\infty}\frac{x^n}{n}$. Find the Intervals of convergence of $f(x)$, $f'(x)$ and $\int f(x)$.

H.W. 4.9 – p.613 #32

Representation of Functions by Power Series

Recall:

Examples:

1) Find a power series for $f(x) = \dfrac{4}{x+2}$, centered at 0.

2) Find a power series for $f(x) = \dfrac{1}{x}$, centered at 1.

3) Find a power series, centered at 0, for $f(x) = \dfrac{3x-1}{x^2-1}$.

4) Find a power series for $f(x) = \ln x$, centered at 1.

5) Find a power series for $g(x) = \arctan x$, centered at 0.

6) Find a power series for $f(x) = \dfrac{4x-7}{2x^2+3x-2}$, centered at 0.

7) Find a power series for $f(x)=\dfrac{4}{4+x^2}$, centered at 0.

8) Find a power series for $f(x)=\dfrac{1}{2x-5}$, centered at 0.

9) Find a power series for $f(x)=\dfrac{3}{4-x}$, centered at $c = -2$.

H.W. 4.10 – p.620
#5,7,8,11,13,15

Taylor and Maclaurin Series

Example #1

Use the function $f(x) = \sin x$ to form the Maclaurin series $\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \ldots\ldots$ and determine the interval of convergence.

Example #2

Show that the Maclaurin series $f(x) = \sin x$ converges to sin *x* for all *x*.

Example #3

Find the Maclaurin series for $f(x) = \sin x^2$.

Example #4

Find the power series for $f(x)=\cos\sqrt{x}$.

Example #5

Find the power series for $f(x)=\sin^2 x$.

Example #6

Use a power series to approximate $\int_0^1 e^{-x^2}\,dx$ with an error of less than 0.01.

Class Work:

Find the power series for $f(x) = \ln x$, centered at $c = 1$.

Find the Maclaurin series for $f(x) = e^{2x}$.

Find the power series for $f(x) = e^{x}$, centered at $c = 1$.

H.W. 4.11 – p.630
#4,18.20,22,23